

## Mean values calculation: A general approach

Let us take  $M$  discrete values of a variable  $u$  given as

$u_1, u_2, \dots, u_M$  with probabilities given by

~~and~~  $P(u_1), P(u_2), \dots, P(u_M)$  respectively

Average value of  $u$  denoted by  $\bar{u}$  is given by

$$\bar{u} = \frac{P(u_1)u_1 + P(u_2)u_2 + \dots + P(u_M)u_M}{P(u_1) + P(u_2) + \dots + P(u_M)}$$

or

$$\bar{u} = \frac{\sum_{i=1}^M P(u_i)u_i}{\sum_{i=1}^M P(u_i)} \quad \text{--- (1)}$$

Instead in place of a variable  $u$  if we take a function  $f$  of variable  $u$ , then the mean value of  $f(u)$  is given by

$$\overline{f(u)} = \frac{\sum_{i=1}^M P(u_i) f(u_i)}{\sum_{i=1}^M P(u_i)} \quad \text{--- (2)}$$

Since we have  $\sum_{i=1}^M P(u_i) = P(u_1) + P(u_2) + \dots + P(u_M)$

it represents the probability that  $u$  assume any one of its possible values and this must be unity.

$$\sum_{i=1}^M P(u_i) = 1.$$

Thus  $\boxed{\overline{f(u)} = \sum_{i=1}^M P(u_i) f(u_i)}$  — (3)

For two functions  $f(u)$  and  $g(u)$  average of sum is given as.

$$\overline{f(u) + g(u)} = \sum_{i=1}^M P(u_i) [f(u_i) + g(u_i)] = \sum_{i=1}^M P(u_i) f(u_i) + \sum_{i=1}^M P(u_i) g(u_i)$$

or  $\boxed{\overline{f(u) + g(u)} = \overline{f(u)} + \overline{g(u)}}$  — (4)

Again if  $c$  is a constant, then

$$\boxed{\overline{cf(u)} = c \cdot \overline{f(u)}}$$
 — (5)

Dispersion of  $u$ !

Deviation of  $u$  from mean value is

$$\Delta u = u - \bar{u}$$

taking average  $\overline{\Delta u} = \overline{u - \bar{u}} = \bar{u} - \bar{u} = 0$

or  $\overline{\Delta u} = 0$

and  $\boxed{(\overline{\Delta u})^2 = \sum_{i=1}^M P(u_i) (u_i - \bar{u})^2 \geq 0}$  — (6)

↑  
Dispersion

$(\overline{\Delta u})^2$  is always  $\geq 0$

$$\overline{(\Delta u)^2} = \overline{(u - \bar{u})^2} = \overline{u^2 - 2u\bar{u} + \bar{u}^2} = \bar{u}^2 - 2\bar{u}\bar{u} + \bar{u}^2$$

or  $\overline{(u - \bar{u})^2} = \bar{u}^2 - 2\bar{u}^2 + \bar{u}^2$

or  $\boxed{\overline{(u - \bar{u})^2} = \bar{u}^2 - \bar{u}^2}$  — (7)

From RHS of (7) we see  $\boxed{\bar{u}^2 \geq \bar{u}^2}$ .